Solutions to Selected Chapter 6 Exercises

6.1 From $\Lambda(T_i) - \Lambda(T_{i-1}) \sim Exponential(1)$, where $\Lambda(T_i) = \lambda T_i$ for the exponential renewal process, we obtain $\Lambda(T_i) - \Lambda(T_{i-1}) = \lambda T_i - \lambda T_{i-1} = \lambda(T_i - T_{i-1}) \sim Exponential(1)$. Consequently, $T_i - T_{i-1} \sim \frac{1}{\lambda} Exponential(1) = Exponential(\lambda)$.

6.2 For failure times $t_1, t_2, \ldots, t_n$, $T_1, T_2 - T_1, \ldots, T_n - T_{n-1}$ are i.i.d. $Gamma(\alpha, \lambda)$. The corresponding likelihood is proportional to

$$[\lambda^\alpha / \Gamma(\alpha)]^n \left(\prod_{i=1}^{n} (t_i - t_{i-1})^{\alpha - 1} \right) \exp(-\lambda t_n)\,,$$

where $t_0 = 0$. Under Type-I censoring at t_c, the corresponding likelihood is proportional to

$$[\lambda^\alpha / \Gamma(\alpha)]^n \left(\prod_{i=1}^{n} (t_i - t_{i-1})^{\alpha - 1} \right) \exp(-\lambda t_n) \int_{t_c - t_n}^{\infty} f(t|\alpha, \lambda) \mathrm{d}t\,,$$

where $f(t|\alpha, \lambda)$ is the $Gamma(\alpha, \lambda)$ probability density function. Type-II censoring is not relevant for a single repairable system.

6.4 A 95% credible interval for κ is (0.473, 1.120), so that the data suggest that there is no need to use the MPLP over the PLP, where κ equals 1. Accounting for the censored observation, $26^{0.4} \approx 4$. Using $K = 4$ equal probability bins, we find that 1.3% of the R^B test statistics exceed the 0.95 quantile of the $ChiSquared(3)$ reference distribution, which suggests no lack of fit.

6.8 If the last failure occurred at t^*, we have $\Lambda(T + t^*) - \Lambda(t^*) \sim Gamma(\kappa, 1)$ so that $R_{t^*}(t) = \mathbf{P}(T > t)$, but

$$T > t \equiv \Lambda^{-1}\left(\Lambda(t*) + X\right) - t^* > t \equiv X > \Lambda(t + t^*) - \Lambda(t^*)\,,$$

where $X \sim Gamma(\kappa, 1)$. Consequently,

$$R_{t^*}(t) = \int_{\Lambda(t+t^*) - \Lambda(t^*)}^{\infty} f(x|\kappa, 1) \mathrm{d}t\,,$$

where $f(x|\kappa, 1)$ is the $Gamma(\kappa, 1)$ probability density function.

6.20 Assuming that uptime $U \sim Gamma(\alpha_U, \lambda_U)$ and downtime $D \sim Gamma(\alpha_D, \lambda_D)$, then the long-run availability $A = \frac{E(U)}{E(U) + E(D)}$ is

$$\frac{\alpha_U / \lambda_U}{\alpha_U / \lambda_U + \alpha_D / \lambda_D}\,.$$